Final Project

EE 345 - Probability and Statistics for Engineers

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The Probability and Statistical Modeling of Jitter in PWM Signals

While evaluating the integrity of Pulse Width Modulated signals (PWM), there can be discrepancies between the rise or fall times of the signal. This change in timing is called Jitter, and when doing bench test reports on signal integrity, it is important to properly characterize the deviations in time the signal may have. With a poorly timed signal, there can be distortion, error accumulation, and reduced system wide reliability which does not allow the circuit to perform at its full capability. Statistical analysis techniques can help identify whether deviations in the signal are caused by random or systematic errors by modeling the observed outcomes.

There are many statistical techniques that can be used to characterize the Jitter of a signal; Normal (Gaussian) random variable, uniform distribution, poisson, and exponential distribution. All of these models have their strengths and weaknesses when evaluating the integrity of a signal, for example, an exponential distribution could be valuable for analyzing high-speed digital systems, where timing deviations are uncommon, but crucial for optimal operation. A poisson process can be used for when there are discrete anomalies in the signal such as missed edges. The most widely used distribution for identifying the integrity is the Dual-Dirac model. This model consists of a random jitter component (Gaussian) and a deterministic component. The random jitter follows a typical gaussian distribution, and the deterministic jitter is “bounded in nature and its probability distribution extends over a finite range from a minimum to a maximum value” [1]. When combining random and deterministic jitter components, we are able to obtain a comprehensive representation of how the signal behaves. In the real world, a PWM signal is neither purely random or purely deterministic, combining the two gives us a clear view into the performance and reliability of our signal.

In addressing whether this problem involves continuous or discrete random variables, the conclusion is that it incorporates both types of random variables. When taking into account random jitter and deterministic jitter, these two factors would consist of continuous random variables because they represent timing deviations which are continuous in nature. Discrete random variables would be used when observing the broad signal performance as a whole, such as the number of times an edge is outside of the acceptable tolerance.

In the analysis of digital systems there are many tools that are useful for engineers to characterize signals. One of these tools is MATLAB. MATLAB has many functions to perform statistical analysis such as fitdist, normfit, histfit, and ksdensity. These functions can help us with fitting and visualizing distributions. An especially useful tool for analysing the Jitter of a PWM signal is the signal processing toolbox. Functions within the toolbox like spectrum, and fft can help us determine whether or not the signal may be systematically or randomly corrupted. Another great tool built into MATLAB is the Simulink platform. Simulink allows the user to go through an interactable block diagram UI to piece signal analysis tools together to perform straightforward and easy analysis.

To gather data to do the statistical analysis, we would first need the proper equipment. The main piece of equipment we would need is an oscilloscope that has a high enough frequency specification to be able to capture the falling or rising edge of the signal. We would also need to use an oscilloscope that has a built-in software tool to automatically capture the duration and location of the falling or rising time. The oscilloscope would also need to be put into persistence mode, this mode would allow the user to see the accumulation of signals in a set time period. With these tools we would be able to gather the necessary data to build a Dual-Dirac Model which would validate the integrity of our digital signal.

After performing the capture and modeling of the PWM signal, we will be able to extract useful data such as the mean jitter, the variance, the standard deviation, the probability that an extreme event would occur, and the jitter spectrum. After the analysis of our signal, we can make engineering decisions based on whether or not the signal falls within our tolerance.

The analysis of PWM and general digital signal integrity is one of the most important benchmarks for how a system will perform as a whole. These methods, when applied thoroughly and properly, can give us a deeper insight into how an electronic circuit operates, as well as, when, how, and why a system might experience failure.

References:

[1] G. Soliman, "The Accuracy of the Gaussian Tail and Dual Dirac Model in Jitter Histogram and Probability Density Functions," in IEEE Transactions on Electromagnetic Compatibility, vol. 64, no. 6, pp. 2207-2217, Dec. 2022, doi: 10.1109/TEMC.2022.3187081.
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